Inherent Costs and Interdependent Impacts of Infrastructure Network Resilience

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Recent studies in system resilience have proposed metrics to understand the ability of systems to recover from a disruptive event, often offering a qualitative treatment of resilience. This work provides a quantitative treatment of resilience and focuses specifically on measuring resilience in infrastructure networks. Inherent cost metrics are introduced: loss of service cost and total network restoration cost. Further, “costs” of network resilience are often shared across multiple infrastructures and industries that rely upon those networks, particularly when such networks become inoperable in the face of disruptive events. As such, this work integrates the quantitative resilience approach with a model describing the regional, multi-industry impacts of a disruptive event to measure the interdependent impacts of network resilience. The approaches discussed in this article are deployed in a case study of an inland waterway transportation network, the Mississippi River Navigation System.

KEY WORDS: Infrastructure; interdependent impacts; networks; resilience

1. INTRODUCTION AND MOTIVATION

Infrastructure networks in today’s global landscape can be characterized as exhibiting the characteristics of many complex and large-scale systems: (i) a large number of interacting components and subsystems, (ii) a large number of decision and state variables, (iii) complicated, complex, and often nonlinear functional relationships, (iv) uncertainty and variability, (v) hierarchical and/or networked interdependencies, (vi) multiple and often conflicting performance objectives, (vii) multiple decisionmakers, and (viii) dynamic changes, among others.1 As such, disruptive events, whether malevolent attacks, natural disasters, manmade accidents, or common cause failures, can have devastating, widespread, and often unpredictable, results.

Consider, for example, the August 2003 U.S. blackout, which “contributed to at least 11 deaths and cost an estimated $6 billion,”2 or the largest blackout in history experienced by India during August 2012, affecting over 600 million people. Against the effects of these events, most research efforts have been devoted to developing traditional measures of protection (hardening)3–5 and policies that can be expensive, degrade typical performance, and are nonreactive. Recent attention has been placed on preparedness, response, and recovery from these events (e.g., in the large-scale homeland security preparedness domain).6 A perspective recently has been collectively referred to in the systems engineering community as designing for resilience, considered an essential component in the design of systems and enterprises.7

The importance of having robust and resilient infrastructure systems has gained the attention of decisionmakers and government officials in the past
decade. Critical infrastructure systems, such as power grids and transportation systems, have been vulnerable to numerous disruptive events, including natural disasters, willful attacks, and accidents. The Department of Homeland Security (DHS) announced a set of grant programs targeting different areas prone to willful attacks or natural disasters, aiming to provide resources helpful in supporting the National Preparedness Goal (NPG) in succeeding in its mission of ensuring “a secure and resilient Nation with the capabilities required across the whole community to prevent, protect against, mitigate, respond to, and recover from the threats and hazards that pose the greatest risk.” Further motivation comes from the resilience paradigm, developed by Henry and Ramirez-Marquez, describing how system performance is affected by the change of the state of the system in the presence of a disruption and throughout the recovery process. It is this resilience paradigm, developed for deterministic and stochastic analyses, that drives the resilience cost metrics in this article.

Figs. 1 and 2 illustrate a general approach for visualizing system performance over time when faced with a disruptive event $e^i$. The states of the system are depicted across the bottom of Fig. 1: the original system state $S_0$ transitions to disrupted state $S_d$ following event $e^i$, and then to recovered state $S_R$ following a recovery effort. The performance of the system is quantified with system service function $\varphi(t)$ (e.g., $\varphi(t)$ could represent commodity flows along a waterway network). In Fig. 1, larger values of $\varphi(t)$ are preferred (therefore, $e^i$ leads to reduced $\varphi(t)$), with the opposite (smaller values of $\varphi(t)$ preferred) depicted in Fig. 2. The reliability, vulnerability, survivability, and recoverability system descriptors, among other details of the state transition, can be found in Baroud et al. Note that recovered system performance need not necessarily be the same as the initial performance prior to the disruptive event. For example, the state of infrastructure following the 2010 earthquake in Haiti may be improved relative to predisruption levels, as part of the recovery activities aiming at helping the infrastructure system regain its functionality might improve the deteriorated state of the system. Further, system performance could fluctuate over time even when no disruption occurs.

We consider resilience to be a time-dependent proportional measure of how the system is performing relative to an as-planned performance level $\varphi(t_0)$, namely, in how different the disrupted performance level $\varphi(t_d)$ is from $\varphi(t_0)$. Given that $R$ is historically reserved for quantifying reliability, resilience is given the notation of $R$ and computed at time $t$ as the ratio of the network performance recovered by time $t$ over the loss of the performance after the disruption occurred (Equation (1)). As such, resilience is a function of the extent of loss experienced at time $t_d$ (or vulnerability) and the speed at which the system recovers (or recoverability).

$$R(t) = \frac{\text{Recovery}(t)}{\text{Loss}(t_d)} \quad (1)$$
A thorough analysis of the recovery dynamics in a network is useful to accurately assess the time and cost needed for a system to regain its functionality and improve the risk-informed decision making in regards to allocating resources after the disruption. Such costs resulting from a disrupted system are due to inherent costs such as (i) the loss of service, (ii) the cost of restoring the system back to a functional state, and other losses such as (iii) the interdependent effects on industries relying on the disrupted system. For example, if the power grid was disrupted in a certain region, (i) the loss of service could be quantified in terms of energy not supplied (megawatts-hours) to a number of customers, (ii) the cost of restoration involves the work crews and equipment needed for repair, and (iii) one of the interdependent impacts could be the loss of production in industries relying on the power generation in that area (and broader to the interdependent relationships outside of the directly impacted area). Other examples include a disruption in certain river links of a waterway transportation network impacting the commodity flow throughout the entire system and in particular, disrupting the functionality of neighboring ports and industries relying on the import and export of commodities through the port.

This work deals with cost and impact metrics aimed at assisting risk managers in accurately identifying and quantifying the multiple costs of a disruptive event in the context of building resilience, with particular emphasis given to decision making under uncertainty. The primary contribution of this article is the integration of (i) the resilience modeling paradigm in Figs. 1 and 2 for a disrupted network with (ii) an economic interdependency model to more accurately quantify the resilience trajectory of indirectly impacted sectors and more effectively make network recovery decisions. Section 2 provides the methodological background with respect to stochastic resilience metrics and interdependency models. Inherent cost metrics are introduced in Section 3, with metrics of interdependent impacts of resilience discussed in Section 4. Section 5 provides an application using a case study of an inland waterway transportation network, the Mississippi River Navigation System. Concluding remarks are given in Section 5.

2. METHODOLOGICAL BACKGROUND

This section details the resilience metrics derived from the depiction of resilience in Figs. 1 and 2, as
well as a risk-informed interdependency model. The background on these methodologies will motivate the development of the inherent cost and interdependent impact metrics proposed thereafter. The discussion of resilience that follows will be centered around networks as opposed to general systems.

2.1. Metrics of Network Resilience

Assume that disruptive event $e^j$ affects the original network state $S_0$ at time $t_e$. The effect on the network is assessed by quantifying the damage to the network service function $\phi(\cdot)$. For example, if the network under study is an inland waterway network, $\phi(\cdot)$ could measure commodity flows, noting that lower service function values are undesirable. After a period of degradation of length $(t_d - t_e)$, the network service function is damaged from its original state, $S_0$ (with corresponding $\phi(t_0)$), to a disrupted state, $S_d$ (with corresponding $\phi(t_d)$). That is, the disruptive effect of such an event is quantified via the analysis of a function $\phi(t)$ describing the behavior of the network as a function of time. After a disrupted state of length $(t_s - t_d)$, the network restoration commences until it reaches a stable system state, $S_f$, with corresponding $\phi(t_f)$. Equation (2) provides a more specific quantification of the value of resilience $\mathcal{R}_\phi(t_r|e^j)$ evaluated at time $t_r \in (t_d, t_f)$.

Set $D$ is the set of possible disruptive events. \[
\mathcal{R}_\phi(t_r|e^j) = \left[ \frac{\phi(t_r|e^j) - \phi(t_d|e^j)}{\phi(t_0) - \phi(t_d|e^j)} \right] \forall e^j \in D \tag{2}
\]

This model enables quantifying and tracking the changes in the network state as a function of time and accurately observing the network response to the recovery strategies employed. Using this metric, decisionmakers can dynamically assess their resilience-building decisions during the aftermath of a disruption. It could also be used as a preparedness decision tool, whereby risk managers decide on investments in vulnerability reduction and/or increased recoverability.

We operationalize $\mathcal{R}_\phi(t_r|e^j)$ from Equation (2) with a set of three metrics describing the time required to achieve different resilience and restoration goals.$^{(28,29,31)}$

First, the metric time to total network restoration, $T_T(e^j)$, records the total time spent from the point when recovery activities commence, at time $t_0$, up to the time when all recovery activities finalize, $T_T(e^j)$. Since these are stochastic metrics, one can calculate the probability that total
system restoration is finished before mission time \( t \) as \( P_R(t) = P(T_R(e^t) \leq t) \).

The second metric, time to full network service resilience, \( T_{\psi(t_0)}(e^t) \), records the total time spent from the point when recovery activities commence, at time \( t_s \), up to the exact time, \( t_f \), when network service is completely restored. From \( T_{\psi(t_0)} \), one can define the probability that network service restoration is finished before mission time \( t_i \) as \( P_T(t_i) = P(T_{\psi(t_0)}(e^t) \leq t_i) \). Note that \( T_{f}(e^t) \geq T_{\psi(t_0)}(e^t) \), or the time at which the network is fully restored, is at least as lengthy as the time until a desired network resilience, say \( \mathcal{I}_{\psi}(t_f|e^t) = 1 \) (though a different target, either better or worse than \( \psi(t_0) \), may be desired), is achieved. For example, flows along a network can occur with full capacity despite not all arcs being restored: full capacity would suggest full network service resilience without all network components being completely restored.

Finally, the metric time to \( a \times 100\% \) resilience, \( T_a(e^t) \), records the total time spent from the point when recovery activities commence, at time \( t_s \), up to the exact time, \( t_o \), when the system service is restored to \( a\psi(t_0) \). From \( T_a \), one can define the probability that network service is restored by \( a \times 100\% \), or \( a\psi(t_0) \), before mission time \( t_o \) as \( P_a(t_o) = P(T_a(e^t) \leq t_o) \). This metric provides a means to compare different recovery strategies, determining which strategy achieves \( a \times 100\% \) resilience the quickest holding resilience constant. Similarly, \( \mathcal{I}_{\psi}(t_o|e^t) \) can be found for different strategies, holding \( t_o \) constant.

A comparison of the different distributions for the different resilience time metrics can help decisionmakers choose the best recovery strategy that would lead to the optimal time to full recovery.

### 2.2. Model of Interdependent Behavior

Adapted from Hernandez-Fajardo and Duenas-Osorio,\(^{32} \) the term cascading effects used throughout this article refers to the inoperability that occurs internally to a network, often induced by flow redistribution resulting from capacity exceedances, and the term interdependent effects refers to the inoperability that occurs external to the network as a result of a disruption to the network, often experienced in other infrastructure and industry sectors. This work integrates the resilience paradigm of Equation (2) with a risk-informed interdependency model to quantify the economic impact of a disruptive event on the interdependent industries that rely on the directly impacted and disrupted network. This section reviews the interdependency model used in this approach.

A widely accepted model for describing the interconnected relationships among infrastructure systems and industry sectors is the Nobel-Prize-winning economic input-output model,\(^{33} \) as shown in Equation (3). For a set of \( n \) infrastructure and industry sectors, \( n \times 1 \) vector \( x \) quantifies production outputs in each sector, \( n \times n \) matrix \( A \) represents the proportional interdependence among sectors (that is, \( Ax \) represents intermediate demand resulting from the production of \( x \)), and \( n \times 1 \) vector \( e \) provides final exogenous consumer demand. As such, Equation (3) describes how changes in consumer demand lead to widespread changes in sector production. The extensive usage of input-output models is due, in part, to the availability of interdependency data describing the interconnected nature of infrastructures and industries in a number of countries,\(^{34} \) including an extensive data collection effort by the U.S. Bureau of Economic Analysis (BEA), which maintains input-output tables at different levels of aggregations.\(^{35} \)

\[
x = Ax + c \Rightarrow x = [I - A]^{-1} c \quad (3)
\]

The input-output model was extended to describe the propagation of inoperability, or the proportional extent to which sectors are not performing in an as-planned manner (e.g., reduced production capability), through several interdependent infrastructure and industry sectors.\(^{36} \) This model, the inoperability input-output model (IIM), is expressed in Equation (4).

\[
q = A^*q + c^* \Rightarrow q = [I - A^*]^{-1} c^* \quad (4)
\]

Vector \( q \) is a vector of infrastructure and industry inoperabilities, proportional reductions in production, describing the extent to which ideal functionality is not realized following a disruptive event. An inoperability of 0 suggests that an industry is operating at normal production levels, while an inoperability of 1 means that the industry is not producing at all. Normalized interdependency matrix \( A^* \) is a modified version of the original BEA-driven \( A \) matrix describing the extent of economic interdependence among a set of infrastructure and industry sectors. \( A^* \) is an interdependency matrix in which every entry represents how much inoperability is contributed by the column industry to the corresponding row industry due to the interdependent nature of industry interactions. The effects of inoperability across
multiple sectors can be expressed with total economic losses, \( Q = x^T q \), or the amount of production made inoperable due to a disruption.

A discrete-time dynamic version of this model calculates the inoperability at any point in time using the recursive formula in Equation (5), quantifying the temporal nature of how inoperability propagates across sectors, then dissipates with recovery.\(^{(37)}\)

The inoperability vector, \( q(t) \), as well as the vector of demand perturbation, \( c^*(t) \), change in time. An \( n \times n \) resilience matrix, \( K \), represents the capability of a certain sector to recover from the disruptive event and reach a desired performance state.

\[
q(t + 1) = (I - K) q(t) + K [A^* q(t) + c^*(t)]
\]

(5)

One means to estimate the entries in matrix \( K \) is in Equation (6), a result of the dynamic version represented in Equation (5) with no temporal demand perturbations.\(^{(37)}\)

In this case, \( K \) is a diagonal matrix with zero nondiagonal entries and \( k_{ii} \) as its diagonal entries. Value \( q_i(0) \) is the initial inoperability experienced in sector \( i \) following a disruptive event, \( q_i(T_i) \) is the desired inoperability state after recovery (assumed to be small but nonzero), which requires \( T_i \) time periods to achieve, and \( a_{ii}^* \) is the diagonal entry in the interdependency matrix. An alternative approach to estimate \( K \) that incorporates the resilience paradigm of Equation (2) is proposed in Section 4.

\[
k_{ii} = \ln \left( \frac{q_i(0)}{q_i(T_i)} \right) / T_i (1 - a_{ii}^*)
\]

(6)

The IIM and its extensions, included among the top 10 contributions to risk analysis from 1980 to 2010,\(^{(38)}\) have been found in a number of risk-related applications, including inventory decision making,\(^{(39,40)}\) transportation closures,\(^{(41,42)}\) workforce disruptions,\(^{(43,44)}\) and electric power outages,\(^{(45,46)}\) among others.

### 3. INHERENT COSTS AND IMPACTS OF NETWORK RESILIENCE

Alluded to previously, two primary inherent costs of network resilience are (i) the cost of lost service and (ii) the cost of restoring the system back to a desired state. Stochastic measures of these two costs are described here.

#### 3.1. Loss of Service Cost

When a disruptive event occurs, a loss in the service of the disrupted network is expected, and the change in the service function quantifies the extent of loss. For example, in the case of a road transportation network, the performance measure could be the traffic flow. Different problems might use different performance measures for the same network, with relevant performance measures determined by the risk manager or the decisionmaker.

A disruptive event is assumed to impact a certain number of components in the network. For road networks, components would be bridges or roads; for inland waterway networks, river links, ports, or dams/locks. Impacted components are assumed to cease functioning for a certain period of time, during which the network is in a disrupted state (Fig. 1). It is assumed that the length of the disrupted state depends on the severity of the event. In Baroud et al.,\(^{(47)}\) the intensity of a disruptive event is assumed to follow a power-law distribution, suggesting that more severe events resulting in longer disrupted state periods of time have a lower probability of occurring.

The power-law distribution has been used in several studies to model extreme events. Studies aimed at determining the probability distribution of the severity of a terrorist attack agree that according to empirical data on worldwide terrorist events from 1968 to 2008, the probability of a terrorist event claiming a number deaths follows a power-law distribution.\(^{(48–50)}\)

Power-law relationships are also used to model the severity of natural disasters such as tornadoes,\(^{(51)}\) other severe weather conditions,\(^{(52)}\) and large earthquakes,\(^{(53)}\) among others. The distribution used to model the severity of a disruptive event is presented in Equation (7), where \( d \) is the number of days the impacted components are not functional (i.e., the period of time during which the system is in a disrupted state). \( C \) is a normalization constant ensuring that the area under the curve of the probability distribution function is equal to 1 and \( \lambda \) is a scaling parameter.

\[
f(d) = Cd^{-\lambda}
\]

(7)

The loss of service cost is then computed as a function of the severity of the disruptive event. If the performance measure is the commodity flow, then the loss of service is the aggregate commodity that was supposed to flow across the disrupted components for the duration of the disruption. Equation (8)
is the performance measure of the network system at the disrupted state.

\[ \varphi(t_d) = \varphi(t_0) - \sum_{i=1}^{m} \varphi_i(t_0) \times \frac{d}{365} \] (8)

\( \varphi_i(t_0) \) is the original performance measure of component \( i \) prior to disruption \( e^i \). It is assumed that annual flow \( \varphi_i(t_0) \) across the network is the sum of individual \( \varphi_i(t_0) \) such that flows are not counted more than once in the sum. If daily commodity flow is available and if a disruption renders \( m \) components completely inoperable for \( d \) days, then the disrupted flow across the network, \( \varphi(t_d) \), can be measured with Equation (8). Since \( d \) follows a probability distribution, simulation techniques (e.g., Monte Carlo) can be used to construct the probability distribution of the loss of service.

### 3.2. Total Network Restoration Costs

Restoring a disrupted network is not only time consuming but costly as a consequence. Careful preparedness strategies and accurate resource allocation should be made to determine the right amount of resources invested at the right time.

To develop a probability distribution for the cost of network restoration, it is assumed that the cost of repairing one component is stochastic. More specifically, we introduce \( C_i(e^i) = C_i^j \) to be the cost of repairing link \( i \) disrupted by event \( e^j \). \( C(e^i) \) defines the vector of costs for all the links disrupted by the same event. The individual component’s restoration cost probability distribution is described in Equation (9). Note that this distribution could also be a function of the severity of the event; the relationship between the component restoration cost distribution and the severity of the event would be case dependent. One particular example will be explored in the case study of this article.

\[ C_i^j = \left\{ C_i^j \right\} \left\{ P(e_r < C_i^j \leq e_r) = \int_{e_r}^{e_r} f(e_i) \, de_i \right\} \] (9)

The total network restoration cost would then be the sum of the individual components’ restoration costs. However, note that taking the sum assumes that component repair is performed in series. To account for potential parallel recovery activities, a constant factor \( \theta_i \) is introduced that would depend on the order in which components are repaired. This factor would have higher values in cases where more components are repaired in parallel and would be multiplied by the individual component’s cost of restoration, as shown in Equation (10). This is similar to the idea of a weighted average, where more weight (in this case higher cost) is given to components repaired in parallel.

\[ C_{total}(e^j) = \sum_i \theta_i C_i^j \] (10)

Similar to the loss of service cost, the total network restoration cost’s probability distribution is constructed by means of simulation, such as Monte Carlo simulation.

Ultimately, decisionmakers would be interested in a metric describing the overall aggregate cost of the entire disruptive event that covers both the loss of service and the cost of restoration. Such a metric can also be computed using simulation of each of the other metrics, provided that they are expressed in the same unit, either dollars, tons of commodity flow, or others.

### 3.3. Interdependent Impacts of Network Resilience

A disruptive event impacting an infrastructure network does not only have impacts on the network itself but also on the surrounding regional infrastructure systems and industries related to and relying upon it. The costs discussed previously are considered to be inherent costs related directly to the disrupted network, and indirect impacts would be losses and costs incurred by infrastructures and industries related to the disrupted system that were not necessarily directly impacted by the disruptive event.

In order to quantify for those indirect losses, an integration of the resilience paradigm in Equation (2) with the discrete-time dynamic interdependency model in Equation (5) is developed. The original interdependency model considered a resilience matrix \( K \) that is held constant throughout the recovery time. That matrix represented the capability of the economy to restore its functionality without taking into consideration the resilience of the underlying disrupted physical infrastructure. Hence, the \( K \) matrix in Equation (5) assumed \( \gamma(t) = 1 \) for the disrupted sector, and is not effectively accounting for the recovery of the underlying physical infrastructure that caused the economic perturbation in the first place.

The approach considers a dynamic version of the resilience matrix that governs the trajectory of interdependent recovery, introducing a matrix whose values are functions of time, \( K(t) \). Further, the
resilience matrix is updated with information regarding the trajectory of resilience as a function of time, as shown in Equation (11). Matrix $\mathbf{K}$ is considered to be a baseline matrix of recovery trajectory whose entries are computed according to Equation (6) that updates the resilience matrix at each point in time with the cumulative resilience of the physically disrupted system. It is also assumed that the perturbation is expressed through a production inoperability; hence, $\mathbf{c}(t) = 0, \forall t$. The new resilience-based dynamic interdependency model is expressed in Equation (12), where $0 < \mathbf{R}_\mathbf{q}(t|e^l) \leq 1$.

$$
\mathbf{K}(t) = \mathbf{K}e^{\mathbf{R}_\mathbf{q}(t|e^l)} \quad (11)
$$

$$
\mathbf{q}(t+1) = (\mathbf{I} - \mathbf{K}(t))\mathbf{q}(t) + \mathbf{K}(t)[\mathbf{A}'\mathbf{q}(t)] \quad (12)
$$

Also, the disrupted system might recover before the rest of the economy does, for which case $\mathbf{R}_\mathbf{q}(t) = 1$ for all $t \geq t_0$, where $t_0$ is the time at which the physically disrupted infrastructure network is recovered. In some other instances, the economy’s recovery does not start until the physically disrupted system is fully recovered, which is the case of a port closure, for example. In others cases in which components of the system (nodes or links) are disrupted, the recovery of the economy would overlap with the recovery of the physically disrupted system; the relation in Equation (11) takes into account the quantitative analysis of such overlap and how the recovery strategy of the physically disrupted system impacts the recovery trajectory of the entire economy.

4. CASE STUDY: INLAND WATERWAY NETWORK

The methodology discussed in this article is applied to an inland waterway transportation network, specifically the Mississippi River Navigation System. The system is a series of river links connecting the different inland ports in the United States. This type of transportation system has a special feature that differentiates it from other regular network systems. Generally, there is usually single point to point access; there is no redundancy due to the nature of the links being a part of the river.

The nation’s economy depends strongly on this waterway network, with nearly 25 billion tons of annual commodity flow circulating through the network. The National Waterway Network (NWN) is composed of a large number of links and nodes. A link represents either a shipping lane or simply a path in open water, and a node could be a facility such as a port, lock, dam, or perhaps another intermodal terminal. The case study analyzes the resilience of a known number of links that might become completely inoperable due to a disruptive event. A U.S. Army Corps of Engineers database was used to construct the 3,046 links of the Mississippi River Navigation System network, as shown in Fig. 3.

The Mississippi River is prone to different types of disruptive events, including periods of drought and flooding. Closing sections of the river impacts the nation’s economy by incurring losses to a large number of industries relying on the shipments that are being delayed, with macrolevel, interdependent losses becoming quite significant. As such, resilience planning for inland waterway networks is of high importance.

4.1. Parameters and Assumptions

A few assumptions were made in this case study. There is one disruptive event impacting four specific links of the river, highlighted in red in Fig. 3 (colors visible in online version). Note that the four links represent in fact 10 segments of the river according to the data of the Army Corps of Engineers. Those segments were condensed into four links due to similar commodity flow capacities and proximity of their location; the event in this case is disrupting operations along a stretch of 141.6 miles of the river located in the area surrounding the port of Catoosa in Oklahoma, resulting in delays in the flow of imports and exports to and from Oklahoma. The individual component restoration cost follows a uniform distribution, with a multiplicative relationship with the severity of the event (Equation (13)). The severity is expressed in terms of days of disrupted state, $d$, and follows a power-law distribution (Equation (14)).

$$
C_i \sim \text{UNI}(0, 1) \times d \quad (13)
$$

$$
P(d) = (1.5 \times d_{\text{min}}^{1.5})d^{-2.5} \quad (14)
$$

Generally, extreme events that are modeled with a power-law distribution result in an estimated scale parameter $\lambda$ ranging between 2 and 3 (as discussed in Section 3.1). It is assumed that $\lambda = 2.5$. It is also assumed that the severity of the disruptive event is bounded with a minimum and a maximum number of days during which the links are disrupted, between $d_{\text{min}} = 5$ and $d_{\text{max}} = 30$ days. The values chosen for the parameters can be thought of as starting values, and sensitivity analysis should be done to observe the
model’s outcome over a specific range for the parameters. For example, $\lambda$ controls the spread of the probability distribution, with larger values resulting in a larger spread of the likelihood of more severe events. Risk-averse decisionmakers might consider larger values to design preparedness options that account for extreme events expressed by the upper tail of the distribution. Also, since the application pertains to a transportation network measured with the number of days transportation flow capability is reduced, there is a need to specify a threshold for the severity of the event to avoid unreasonable impacts that alter the preparedness decision and resource allocation without a significant tradeoff of protection.

In the interdependency model, industry inoperability is assumed to be the result of imports failing to reach the port and causing a shortage in the material needed to produce in the industries relying on that commodity. As such, inoperability in industry $i$ is then the ratio of its imports to its total production output, as an industry can only be as productive as its most disrupted supplier.$^{41,42}$ Equation (15) illustrates this thought process as the maximum initial inoperability experienced in industry $i$. The yearly estimate of imports from industry $j$ is $m_j$, the total production output for industry $j$ is $x_j$, and the number of industries that typically circulate on the disrupted links is $h$. This maximum perturbation corresponds to a disruption lasting for a year; therefore, for a disruption of duration $d$ days, the initial inoperability is computed in Equation (16). This approach has been used in previous interdependent impact analyses for disruption of inland waterway ports.$^{42,57}$

$$q_{i, \text{max}} = \max \left( \frac{m_1}{x_1}, \ldots, \frac{m_j}{x_j}, \ldots, \frac{m_h}{x_h} \right)$$ (15)

$$q_i (0) = \left( \frac{d}{365} \right) q_{i, \text{max}}$$ (16)

Commodity flow data for each link in the Mississippi River Navigation System are provided by the U.S. Army Corps of Engineers, composed of the
Table I. Annual Commodity Flow (in Tons) Across the Five Links Chosen from the Mississippi Navigational System Network

<table>
<thead>
<tr>
<th>Link ID</th>
<th>Total Annual Commodity Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>5,007,904</td>
</tr>
<tr>
<td>600</td>
<td>2,409,925</td>
</tr>
<tr>
<td>709</td>
<td>4,766,979</td>
</tr>
<tr>
<td>810</td>
<td>6,236,462</td>
</tr>
</tbody>
</table>

Table II. Four Recovery Sets Considered for the Restoration of Disrupted Waterway Links

<table>
<thead>
<tr>
<th>Recovery Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>Repair links in series in the order: 500–600–709–810</td>
</tr>
<tr>
<td>W2</td>
<td>Repair link 500 first, then 600 and 709 in parallel in the second order, and 810 in the third order</td>
</tr>
<tr>
<td>W3</td>
<td>Repair link 500 first, then links 600, 709, and 810 in parallel in the second order</td>
</tr>
</tbody>
</table>

yearly tonnage of commodity flow, with the commodity flows of five chosen links provided in Table I. We consider the daily commodity flow to be the annual flow divided by 365 days.

We examine three possible recovery activities sets, described in Table II, for the four disrupted links with IDs 500, 600, 709, and 810.

4.2. Resilience and Restoration Time Results

We first observe one possible realization for the resilience trajectory over time. One observation is drawn from a triangular distribution with parameters randomly selected for each link, and we compute the resilience at each point in time using Equation (2) based on the three strategies in Table II. Note the difference in the time required to achieve full network resilience, portrayed in Fig. 4. W1 requires approximately 15 additional time units of recovery activities when compared with W2, and almost 35 additional time units when compared with W3. Also, W2 and W3 differ by 20 time units. Clearly, as more links are repaired in parallel, the trajectory toward a fully resilient network tends to be faster, leading to a shorter recovery time. Note that the step function is used in this article to describe the resilience over time, while other types of linear and nonlinear functions can be investigated.\(^{(58)}\)

To more effectively represent the variability in the underlying model parameters, we simulate 2,000 scenarios of possible disruptive events and look at the cumulative distribution function (cdf) of time to full restoration under the three strategies. The conclusions align with the observations from Fig. 4 as more links are repaired in parallel, the overall time to full network restoration decreases. Fig. 5 suggests that W3 generally dominates the other two strategies for much of the length of the cdfs. For example, the probability that full network restoration occurs before 20 days is approximately 0.2 for W1, 0.60 for W2, and 0.80 for W3. Only the average point estimate of the simulated times to full recovery suggests a similar outcome. There is, on average, a decrease of 9.5 time units between W1 and W2, 15.4 between W1 and W3, and 5.9 between W2 and W3. It is clear that going from strategy W1 to strategy W2 has a greater impact, almost double, than going from strategy W2 to strategy W3. To make a better decision regarding which strategy to choose, risk managers look at other objectives, such as costs and interdependent impacts, to determine the tradeoff between the strategies.

4.3. Inherent Cost Results

Mentioned previously, the cost of a disruptive event has several dimensions: the loss of service cost, the cost of network restoration, and the cost incurred by interdependent impacts. Using the 2,000 simulations of possible scenarios for disruptive events, we construct the probability distribution function (pdf) and cumulative probability distribution (cdf) for the two cost metrics: the loss of service cost in Figs. 6(A) and (C), and the total restoration cost in Figs. 6(B) and (d) for recovery strategy W1. Given the nature of the power-law distribution, smaller values of cost are more likely to occur with extreme values becoming increasingly less likely. Also, note the difference in the units of the cost between the loss of service cost and the network restoration cost: the loss of service cost is measured in terms of tons of commodity flow and the restoration cost is measured in thousands of dollars. To commensurate costs, the commodity flow in tons would be converted into dollar amounts (if available) before adding the two random variables and generating a distribution for the total cost of the disruption.

The loss of service cost depends solely on the severity of the event and is not impacted by the difference in the recovery strategies. However, from Equation (10), different strategies have different costs that increase as more links are repaired in parallel, which impacts the cost incurred by the recovery activities. The individual cost for repairing each
link is multiplied by the number of links that are being repaired in parallel with this particular link. Given the manner in which this is modeled in Equation (10), Fig. 7 suggests that W3 incurs the highest cost with three out of the four disrupted links being repaired in parallel, while W1 has the lowest cost with all links being repaired in series. Also, note that going from W2 to W3 involves a larger investment than going from W1 to W2. In fact, on average, W2 costs more than W1 by 9.4 cost units and by 19.2 for W3, this will help decisionmakers in choosing a recovery strategy by assessing the tradeoff between the strategies.

4.4. Interdependent Impact Results

We now examine the interdependent impacts of a disruptive event in the waterway network on the industries relying on the commodities flowing on the network, in light of the resilience quantification and the three strategies considered above.

Table AI in the Appendix lists the combined estimates for the annual exports and imports in tons of the eight primary industries using the Port of Catoosa among the states that do the most commerce using the port. Sixty-three BEA sectors, listed in Table AII of the Appendix, were considered in the multi-industry impact analysis. Fig. 8
Fig. 6. Approximate pdf results for 2,000 simulations of (A) the loss of service cost and (B) the network restoration cost, along with their respect cdfs in (C) and (D).

Fig. 7. Cumulative distribution function for the network restoration cost.

depicts the relationship among (i) network resilience, \( \mathcal{R}_\nu (t | e_j) \), represented as a dark line, and (ii) inoperability of a selected number of industries, \( q_i (t) \), represented with lighter gray lines. The selected industries are those with imports flowing primarily on the disrupted links: food and beverage and tobacco products (FBT), petroleum and coal products (PC), chemical products (CH), nonmetallic mineral
Fig. 8. Resilience (black line, right vertical axis) and sector inoperabilities (gray lines, left vertical axis) over 50 time periods for the three recovery strategies.
products (NMM), primary metal products (PM), and fabricated metal products (FM). Originally considered over 120 days in Fig. 5, we reduce the number of days along the horizontal axis in Fig. 8 to better illustrate (i) when $\mathbb{I}_q^t (e_j)$ approaches 1, or full network resilience, and (ii) when $q_i (t)$ approaches 0 for the selected industries. Inoperability is depicted on the vertical axis on the left, while resilience, ranging from 0 to 1, appears on the vertical axis on the right.

Naturally, resilience as measured by $\phi (t)$ is increasing with time, while inoperability is decreasing. For these particular recovery examples, the individual sectors recover faster with strategies W2 and W3 with steeper decreasing trajectories for the individual industries inoperability. This is due to the parallel recovery activities in strategies W2 and W3 speeding the recovery of the disrupted network and hence resulting in a faster recovery for the rest of the economy. Also, note that for recovery strategies W1 and W2, the full recovery of the economic sectors almost aligns with the full recovery of the disrupted system, while for strategy W3, the disrupted network recovered very fast and the economy fully recovers shortly after that. Finally, for one of the sectors, the inoperability continues to increase at the beginning, primarily due to the relatively high initial inoperability resulting in an infinitesimal impact of the resilience matrix on the recovery process. Such observation might trigger a need for system hardening or extra resources if decisionmakers want this sector to start to recover sooner.

When economic losses are aggregated across time, that is, the cumulative effect of inoperability multiplied by the as-planned output of each industry, the effect of recovery strategy can impact industries in different ways. The average economic losses from 2,000 simulations are depicted in Fig. 9 for the three strategies. The same pattern is seen across all three strategies: the petroleum and coal products industry experiences the most economic losses by far, with fabricated metal products next, and the chemical products industry affected the least in economic terms. However, the extent to which these industries are impacted does depend on the strategy, with W3 resulting in the fewest losses across industries. Such a breakdown can point a decisionmaker in the appropriate direction when patterns point to key industries.
Figs. 8 and 9 provide average behavior when \( d \), the number of days the waterway links are disrupted, is treated as a random variable. Fig. 10 focuses on four particular disruption lengths: \( d = 5 \), \( d = 10 \), \( d = 15 \), and \( d = 20 \). Behavior of economic losses across all industries is depicted with the blue curve, corresponding to the left axis. The trajectory of resilience over time is depicted with the green curve and the right axis ranging from 0 to 1. Note that resilience reaches 1 in a shorter time as we go from W1 to W3 for one particular disruptive scenario, and it takes longer for scenarios with larger impacts. Also, economic losses increase as the disruptive event’s severity increases, but they decrease as recovery strategies become faster. Total economic losses range roughly from $45 to $1,200 million for disruptions lasting from 5 to 20 days under strategy W1, for example. The decrease in total economic losses for faster recovery strategies can be observed by examining plots of the same disrupted scenario as the y-axis of the total economic loss is the same across different recovery strategies. Hence, observations in the plots below align with the conclusions from Figs. 8 and 9. A faster recovery strategy means that at a certain point in time, the resilience of the disrupted system is larger than one of a slower recovery strategy, and this results in a more resilient industries being able to recover faster and this is portrayed by larger entries in the resilience matrix, according to Equation (11).

In order to compare the three recovery strategies W1, W2, and W3, interdependent impacts for a particular disruption scenario (\( d = 10 \)) are examined further. Consider first the total economic loss, an aggregation of the losses incurred across all sectors at each point in time from the disruption through recovery. This function is cumulative; therefore,
naturally exhibits an increasing pattern, as illustrated in Fig. 11. Note that the total economic loss is assessed to be larger under strategy W1, and W3 results in the lowest estimate for the total economic loss. Note that the different recovery strategies are impacting the interdependency model through (i) the resilience trajectory and (ii) the time to full recovery. A faster recovery of the disrupted system leads to less total economic losses incurred as the economy can recover faster and more effectively. However, the faster the strategy is, the more costly it is.

Rather than a multi-industry economic loss perspective, focus could be given to a particular industry. In particular, we look at the industries whose commodities flow along the disrupted waterway links: food and beverage and tobacco products (FBT), petroleum and coal products (PC), chemical products (CH), nonmetallic mineral products (NMM), primary metal products (PM), and fabricated metal products (FM). Figs. 12 and 13 are plots of the difference in the inoperability of the individual sectors when considering strategies W1 and W2, and strategies W2 and W3, respectively.

In both cases, the sector of primary metal products is the most impacted by the change of strategies, while the rest of the sectors have comparable and a smaller change in the estimated inoperability. Note that all observe the same pattern in the difference in inoperability over the time to full network recovery. This pattern suggests that opting for a faster
recovery strategy is not always beneficial. After time $t = 15$, the magnitude of the tradeoff of switching to W2 from W1 starts to decrease, suggesting that this might be a good time for decisionmakers to switch back to a cheaper strategy. A similar conclusion can be drawn from the comparison between W3 and W2, with an overall much smaller difference in the inoperability between the two strategies.

Fig. 14 shows the inoperability trajectory for the sector of primary metal products under the three different recovery strategies. The results shown comply with Fig. 11, total economic loss under the three recovery strategies. Adopting strategy W1 results in an estimation of a largest inoperability, while W3 results in a much steeper decreasing trajectory for the inoperability of the primary metal products sector.

With such an extensive analysis, decisionmakers have a range of metrics to consider in order to choose the best recovery strategy that balances cost and risk management. While the example above considered three recovery strategies, a more exhaustive set of strategies could be considered and compared with a stochastic ordering technique (e.g., the Copeland score method\(^{(29)}\)). Using the metrics above, decisionmakers can examine the possible strategies that minimize the cost (W1), or minimize the total economic losses (W3), or a combination of the two based on how a strategy is significantly better than the other over time.
5. CONCLUDING REMARKS

Risk managers preparing for all such sources of disruptive events must plan for the interconnected relationship of infrastructure networks with the industries that rely upon them. Most work in infrastructure networks addresses esoteric graph theoretic measures of topology (e.g., centrality, betweenness) that may provide little insight for decision making.\(^{(59)}\) However, the ultimate usefulness of understanding interdependent effects for a sustainment decisionmaker is not just a descriptor of physical damage, but of economic interruption (the result of a lack of functionality).\(^{(60,61)}\) That is, the benefit of physical models of interdependence is lost unless they ultimately translate into (i) dollars of losses incurred, and (ii) extent of and duration of system inoperability.

This article, which progresses from previous work by the authors, presents a stochastic approach to compute three metrics of the resilience of an infrastructure network following a disruption: (i) the loss of service cost, (ii) the total network restoration cost, and (iii) the cost of interdependent impacts. These three metrics extend from prior work in stochastic network resilience.\(^{(29,31)}\) The first two metrics are modeled using simulation of probability distributions. The economic impacts of a disruptive event is a well-studied topic,\(^{(22,62-65)}\) but the third metric developed here represents a first step in measuring the broader multi-industry impacts of resilience in infrastructure networks, integrating a network resilience model and an economic interdependency model.

A case study involving the disruption of links on a waterway infrastructure network illustrates these concepts and results demonstrate the importance of considering such measures in risk-informed decision-making problems. Incorporating resilience in the interdependency model is helpful to accurately assess the patterns in the cost metrics over time as the system is recovering. Strategies differ in their cost of implementation and interdependent impact, allowing a decisionmaker to understand tradeoffs among different objectives. In this particular example, the petroleum and coal products industry was most impacted, on average, by the disruption, measured by a stochastic duration of commodity flow stoppage, and the interdependent inoperability in the primary metal products industry was most affected by the change in recovery strategy.

Future work includes the resilience-based analysis of a more extensive set of disruptive scenarios and recovery strategies, as well as the exploration of network examples where disruptive events are not as localized as inland waterways. Further, particularly for the inland waterway case study, further validation of the model and its usefulness in guiding network recovery strategies through conversations with stakeholders (e.g., U.S. Army Corps of Engineers) is sought.

APPENDIX

<table>
<thead>
<tr>
<th>Industry</th>
<th>Food/ beverage</th>
<th>Petro/ coal</th>
<th>Chem products</th>
<th>Nonmetallic</th>
<th>Primary metals</th>
<th>Fab metal</th>
<th>Machinery</th>
<th>Misc. manuf</th>
<th>Total</th>
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<td>38,267</td>
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<td>677,753</td>
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<td>6,426</td>
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<td>Imports to LA</td>
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<td>7,210</td>
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<td>11,479</td>
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<tr>
<td>Imports to MS</td>
<td>8,488</td>
<td>45,928</td>
<td>438,338</td>
<td>86,090</td>
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<td>585,761</td>
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<td>Imports to OH</td>
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<td>5,377</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>5,377</td>
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<tr>
<td>Total</td>
<td>602,926</td>
<td>193,952</td>
<td>748,825</td>
<td>45,887</td>
<td>289,557</td>
<td>23,485</td>
<td>11,436</td>
<td>1,600</td>
<td>1,917,668</td>
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</table>

Table A1. Estimates of Annual Tonnage of Exports and Import Through the Port of Catoosa for 2007
Table AII. The 63 BEA Industry and Infrastructure Sectors Comprising the Illustrative Examples

<table>
<thead>
<tr>
<th>Industry</th>
<th>Description</th>
<th>Industry</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Farms</td>
<td>33</td>
<td>Transit and ground passenger transportation</td>
</tr>
<tr>
<td>2</td>
<td>Forestry, fishing, and related activities</td>
<td>34</td>
<td>Pipeline transportation</td>
</tr>
<tr>
<td>3</td>
<td>Oil and gas extraction</td>
<td>35</td>
<td>Other transportation and support activities</td>
</tr>
<tr>
<td>4</td>
<td>Mining, except oil and gas</td>
<td>36</td>
<td>Warehousing and storage</td>
</tr>
<tr>
<td>5</td>
<td>Support activities for mining</td>
<td>37</td>
<td>Publishing industries (includes software)</td>
</tr>
<tr>
<td>6</td>
<td>Utilities</td>
<td>38</td>
<td>Motion picture and sound recording industries</td>
</tr>
<tr>
<td>7</td>
<td>Construction</td>
<td>39</td>
<td>Broadcasting and telecommunications</td>
</tr>
<tr>
<td>8</td>
<td>Food and beverage and tobacco products</td>
<td>40</td>
<td>Information and data processing services</td>
</tr>
<tr>
<td>9</td>
<td>Textile mills and textile product mills</td>
<td>41</td>
<td>Federal Reserve banks, credit intermediation, and related activities</td>
</tr>
<tr>
<td>10</td>
<td>Apparel and leather and allied products</td>
<td>42</td>
<td>Securities, commodity contracts, and investments</td>
</tr>
<tr>
<td>11</td>
<td>Wood products</td>
<td>43</td>
<td>Insurance carriers and related activities</td>
</tr>
<tr>
<td>12</td>
<td>Paper products</td>
<td>44</td>
<td>Funds, trusts, and other financial vehicles</td>
</tr>
<tr>
<td>13</td>
<td>Printing and related support activities</td>
<td>45</td>
<td>Real estate</td>
</tr>
<tr>
<td>14</td>
<td>Petroleum and coal products</td>
<td>46</td>
<td>Rental and leasing services and lessors of intangible assets</td>
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<tr>
<td>15</td>
<td>Chemical products</td>
<td>47</td>
<td>Legal services</td>
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<tr>
<td>16</td>
<td>Plastics and rubber products</td>
<td>48</td>
<td>Miscellaneous professional, scientific, and technical services</td>
</tr>
<tr>
<td>17</td>
<td>Nonmetallic mineral products</td>
<td>49</td>
<td>Computer systems design and related services</td>
</tr>
<tr>
<td>18</td>
<td>Primary metals</td>
<td>50</td>
<td>Management of companies and enterprises</td>
</tr>
<tr>
<td>19</td>
<td>Fabricated metal products</td>
<td>51</td>
<td>Administrative and support services</td>
</tr>
<tr>
<td>20</td>
<td>Machinery</td>
<td>52</td>
<td>Waste management and remediation services</td>
</tr>
<tr>
<td>21</td>
<td>Computer and electronic products</td>
<td>53</td>
<td>Educational services</td>
</tr>
<tr>
<td>22</td>
<td>Electrical equipment, appliances, and components</td>
<td>54</td>
<td>Ambulatory health-care services</td>
</tr>
<tr>
<td>23</td>
<td>Motor vehicles, bodies and trailers, and parts</td>
<td>55</td>
<td>Hospitals and nursing and residential care facilities</td>
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<tr>
<td>24</td>
<td>Other transportation equipment</td>
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<td>Social assistance</td>
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<td>25</td>
<td>Furniture and related products</td>
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<td>Performing arts, spectator sports, museums, and related activities</td>
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<td>26</td>
<td>Miscellaneous manufacturing</td>
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<td>Amusements, gambling, and recreation industries</td>
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<td>Wholesale trade</td>
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<td>Air transportation</td>
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<td>Other services, except government</td>
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<td>Rail transportation</td>
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<td>Federal government</td>
</tr>
<tr>
<td>31</td>
<td>Water transportation</td>
<td>63</td>
<td>State and local government</td>
</tr>
<tr>
<td>32</td>
<td>Truck transportation</td>
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REFERENCES