Problem of the Week
Sept. 24, 2003

You have a rectangular chocolate bar marked into $m \times n$ squares, and you wish to break up the bar into its constituent squares. At each step you may break one piece along any of its marked vertical or horizontal lines. Prove that every method of breaking the bar up finishes in the same number of steps.

This old chestnut appears in a paper by Peter Winkler entitled "Five Algorithmic Puzzles," which is available online at

http://cm.bell-labs.com/cm/ms/who/pw/papers/algor.ps

Dr. Winkler's homepage is the same address minus the papers/algor.ps part. As he mentions in the paper where the problem is given, "This one has been known to stump some very high-powered mathematicians for as much as a full day, until the light finally dawns amid groans and beatings of the head against the wall."

The point is that, whatever shape the bar may be and however it may may be marked, you are only allowed each turn to take one piece and break it into two pieces! Thus you start with one piece and break that in some way and now have two pieces; you now select one of those two, break it somehow, and you now have three pieces; take one of those three pieces, break it, and you now have four, etc. *Each turn you create one more piece, regardless.* Since you wish to end up with mn pieces and started with one piece, you will finish in $mn - 1$ steps, regardless. The grid pattern of the breaks you may make does not matter and serves only as a red herring to throw you off of the track.

By the way, the paper cited has four more problems in it that are if anything more clever than this one. Those problems are actually worked out in the paper, and in fact the paper's intent is to take the "trickery" out of problems like these. The paper presents a very logical and practical approach to algorithmic problems of this type, and I recommend it and think you could find it entertaining and useful.