

**Simplify the expression**

$$\sqrt{3 - \sqrt{5}} + \sqrt{4 + \sqrt{7}} + \sqrt{6 - \sqrt{35}}$$

It turns out that the expression above is equivalent to  $\sqrt{14}$ . It's apparent that an alternative way of writing expressions of the form  $\sqrt{x \pm \sqrt{y}}$  would be useful here. That turns out to be something that's fairly easy to get. Notice that if we start with something of the form

$$\sqrt{x \pm \sqrt{y}} = \sqrt{a} \pm \sqrt{b}$$

Then squaring both sides gives

$$x \pm \sqrt{y} = a + b \pm 2\sqrt{ab}$$

So letting  $x = a + b$  and  $y = 4ab$  and solving that system for  $a$  and  $b$  will give us the desired simplification. Doing so we see that

$$\begin{aligned}\sqrt{3 - \sqrt{5}} &= \sqrt{\frac{5}{2}} - \sqrt{\frac{1}{2}} \\ \sqrt{4 + \sqrt{7}} &= \sqrt{\frac{7}{2}} + \sqrt{\frac{1}{2}} \\ \sqrt{6 - \sqrt{35}} &= \sqrt{\frac{7}{2}} - \sqrt{\frac{5}{2}}\end{aligned}$$

and so our expression is equal to  $2\sqrt{\frac{7}{2}} = \sqrt{14}$ .

This problem has previously appeared as the September 2000 "Ponder This" Challenge at the IBM Research website. The general website for IBM Research is

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