

**Nine points are selected from inside the unit cube. Show that at least two of the nine points are less than  $\frac{\sqrt{3}}{2}$  apart.**

We may assume that the cube is centered at the origin. Then the  $xy$ -plane, the  $xz$ -plane, and the  $yz$ -plane divide the cube into eight equal, smaller “subcubes.” Since we have nine points, at least two of them must lie in the same subcube. The maximum distance apart these two points can be is if they lie at opposite corners of the subcube, and opposite corners of the subcube are  $\frac{\sqrt{3}}{2}$  apart. The distance must be less than this since we stipulated that the nine points are selected from *inside* the original unit cube.