

You are on a team of three people that is given the following challenge on a reality tv show. The host of the show will randomly place either a red or blue hat on everybody's heads in such a way that each contestant will be able to see the other players' hats but not their own. The host will then give each contestant the option to secretly write down a guess about their own hat color. The contestants are not forced to guess; they may pass. The team wins if any player correctly guesses the color of the hat on their own head, but loses if any player guesses incorrectly or if all three players pass.

It is fairly obvious that if the other members of the team pass and allow you to guess randomly then your team will have a 50% chance of winning. Your team has some time before the game begins to discuss strategy. Can you come up with a strategy that gives a better chance of winning?

There is in fact a better strategy, one that will give your team a 75% chance of winning: If any member of the team sees two hats of the same color on the other contestants' heads, he (or she) should guess the opposite color. If a team member sees two different hats on the other contestants' heads, he (or she) should pass.

To see when this works, examine when this strategy will fail. Since each person can have one of two kinds of hats there are only eight possible ways for the host to put hats on the contestants' heads. Those possibilities are

$$\begin{array}{cccc} (R, R, R) & (R, B, R) & (B, R, R) & (B, B, R) \\ (R, R, B) & (R, B, B) & (B, R, B) & (B, B, B) \end{array}$$

where R in the first spot means the first contestant got a red hat, etc. The only hat configurations listed above in which the strategy would fail are the configurations (R, R, R) and (B, B, B) , where everyone has the same hat color. In the other cases, the two team members who have the same hat color will pass while the remaining team member will guess correctly. This means that the strategy succeeds in 6 out of the 8 possibilities, and since each possibility is equally likely, we have an overall success rate of 75%.

A very good exposition of this problem and its generalizations can be found in Winter 2002 edition of *The Mathematical Intelligencer*, in an article entitled "Hat Tricks." The problem was introduced in the 1998 PhD thesis of Todd Ebert, where it was stated in terms of a number of prisoners and a warden. As the article notes, "Good problems travel fast," and the problem took on a life of its own, even eventually landing in *The New York Times* as an example of team behavior outperforming all possible individualistic behaviors.