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The answer is on the next page.

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Let's let N_k denote the number of ways to climb k stairs in the manner described. (So we're looking for N_{10} .) Notice that for $k \geq 4$ there are 3 "moves" one can do for your first step: you can climb 1,2, or 3 stairs. If you climb 1 stair then there are N_{k-1} ways to finish; if you climb 2 stairs there are N_{k-2} ways to finish; and if you climb 3 stairs there are N_{k-3} ways to finish. Thus:

$$N_k = N_{k-1} + N_{k-2} + N_{k-3}$$

Well, it's pretty easy to see that for the $k < 4$ we have $N_1 = 1$, $N_2 = 2$ and $N_3 = 4$, so using the above we can calculate N_{10} using brute force.

$$\begin{aligned} N_4 &= N_3 + N_2 + N_1 = 4 + 2 + 1 = 7 \\ N_5 &= N_4 + N_3 + N_2 = 7 + 4 + 2 = 13 \\ N_6 &= N_5 + N_4 + N_3 = 13 + 7 + 4 = 24 \\ N_7 &= N_6 + N_5 + N_4 = 24 + 13 + 7 = 44 \\ N_8 &= N_7 + N_6 + N_5 = 44 + 24 + 13 = 81 \\ N_9 &= N_8 + N_7 + N_6 = 81 + 44 + 24 = 149 \\ N_{10} &= N_9 + N_8 + N_7 = 149 + 81 + 44 = 274 \end{aligned}$$

There are 274 ways to climb the stairs.