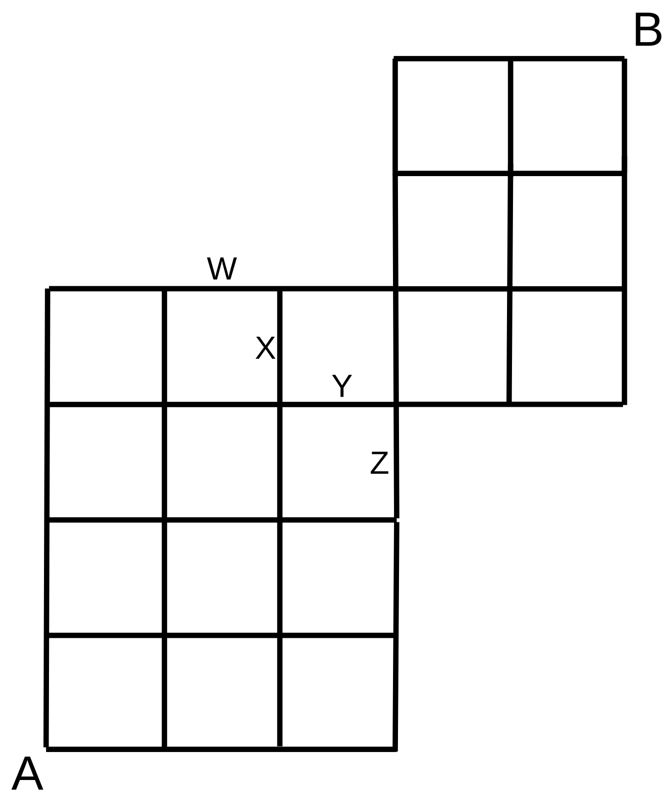
The figure below shows a map of Squareville, where each city block is of the same length. Two friends, Alexandra and Brianna, live at the corners marked A and B, respectively.

The two friends start walking toward each other's house, leaving at the same time, walking with the same speed, and independently choosing a minimum distance path to the other's house. (Thus Alexandra always moves either up or right, and Brianna always moves down or left.) What is the probability that they will meet?



I've marked four blocks on the above grid with letters W, X, Y and Z. Obviously we are singling these blocks out because they are important, and they are—these are the spots that are exactly halfway in between points A and B, and since the girls are moving at equal speeds, these are the only blocks where a meeting can happen. With this fact in mind, our question essentially boils down to this: If A has selected a path to B and B has selected a path to A, what is the probability that A's path and B's path share one of these blocks? (Every path from A to B must pass through one of these blocks.)

Now any path from A to B is also a path from B to A (just run it in reverse), so really we just need to figure out the paths in one direction. Let's first tackle an easier case that will help us solve the real problem: How many ways are there to move from the lower corner to the upper corner of a *full* grid, i.e. a rectangle m blocks wide and n blocks tall? We assume of course that, like in the problem, you always have to move up or to the right. Obviously the trip will take $m+n$ steps, and m of those steps will have to be moves to the right, and the remaining n steps will be moves upwards. This is equivalent to filling $m+n$ blanks with the m R's and n U's, and there are $\binom{m+n}{m} = \binom{m+n}{n}$ ways to do that.

Now we are equipped to count the paths from A to B quickly.

- There are $\binom{5}{1} = 5$ ways to get to the beginning of block W. After block W we must move to the right, and after that we have $\binom{4}{2} = 6$ ways to finish our trip. Total: $5 \cdot 6 = 30$ paths.
- There are $\binom{5}{2} = 10$ ways to get to the beginning of block X. After block X we must move to the right, and after that we have $\binom{4}{2} = 6$ ways to finish our trip. Total: $10 \cdot 6 = 60$ paths.
- There are $\binom{5}{2} = 10$ ways to get to the beginning of block Y, and after block Y there are $\binom{5}{2} = 10$ ways to finish the trip. Total: $10 \cdot 10 = 100$ paths.
- There are $\binom{5}{3} = 10$ ways to get to the beginning of block Z, and after block Z there are $\binom{5}{2} = 10$ ways to finish the trip. Total: $10 \cdot 10 = 100$.

So there are a total of 290 paths from A to B. Even better, we have a breakdown of how many of these 290 total go through each possible meeting block. We can now answer the question. To save space, let the symbol $A \to W$ mean "A's path goes through block W", and so on for B and the other special blocks. We have:

$$\begin{aligned}
P(meeting) &= P(A \to W)P(B \to W) + P(A \to X)P(B \to X) \\
&\qquad + P(A \to Y)P(B \to Y) + P(A \to Z)P(B \to Z) \\
&= \left(\frac{30}{290}\right)\left(\frac{30}{290}\right) + \left(\frac{60}{290}\right)\left(\frac{60}{290}\right) + \left(\frac{100}{290}\right)\left(\frac{100}{290}\right) + \left(\frac{100}{290}\right)\left(\frac{100}{290}\right) \\
&= \frac{245}{841}
\end{aligned}$$

There is a $\frac{245}{841} \approx 29.13$ percent chance the friends will meet up.