

It is helpful to first consider the problem of finding the probability that a single drunk will return to the origin. Because there are two possible outcomes on each trial, the probability of the drunk's final location is given by a binomial distribution:

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}$$

Where n is the number of steps to the right and p is the probability of taking a step to the right. If the number of steps to the right and to the left is equal, then the drunk will return to the origin, i.e., when $n = N/2$. And because right and steps occur with equal probability, $p = 1/2$. Then the probability the drunk returns to the origin is:

$$P = \frac{N!}{[(N/2)!]^2} \frac{1}{2^N}$$

Now to consider the problem of two drunks meeting after N steps. It will help to rethink the problem. We can consider the first drunk to be the "origin", and the second as we did in the previous problem. Now we simply keep track of the relative position.

Now, if the first drunk takes a step to the right, we can consider it the same as a step to the left for the second drunk with the first drunk remaining stationary (and vice versa). So, essentially, the problem is the same as asking the probability that a drunk return to the origin after $2N$ steps. Therefore, the probability the drunks meet after N steps is:

$$P = \frac{(2N)!}{(N!)^2} \frac{1}{2^{2N}}$$