Quantum Control of an Open System: Stochastic Approach
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Introduction

- For an open quantum system, mutual interactions due to the coupling between the system and environment are inevitable and can result in very complex reduced dynamics including dissipation, fluctuation, decoherence and disentanglement.
- We use the quantum state diffusion (QSD) equation to implement the Uhrig dynamical decoupling (UDD) to a three-level quantum system coupled to a practical non-Markovian reservoir.
- We demonstrate that an unknown state of the three-level quantum system can be universally protected against both colored phase and amplitude noises when the control-pulse sequences and control operators are properly designed.
- We derive the exact time-local non-Markovian stochastic state diffusion equation to numerically simulate the three-level open quantum system under the UDD control.
- We show how the control efficacy depends on the environment memory time and the designed time points of applied control pulses.

Qudit Dissipative Model

The total Hamiltonian describes a three-level quantum system coupled to a dissipative noise can be written as:

$$H = H_0 + P_1 (J_1 - J_1^0) + P_2 (J_2 - J_2^0)$$

where the two control operators for qudit dissipative decoupling are given by:

$$P^{(1)}_1 = \sum_m |m\rangle \langle m|$$

$$P^{(2)}_2 = \sum_m |m\rangle \langle m|$$

by rotating the reference frame twice, we get the total effective Hamiltonian in the new Toggling frame:

$$\tilde{P} = \sum_{m,n} \langle m| \langle m| \tilde{P} |m\rangle |n\rangle \langle n|$$

where the control operator for qudit pure dephasing model is:

$$P = \sum_{m,n} \langle m| \langle m| \tilde{P} |m\rangle |n\rangle \langle n|$$

The time points for the applied pulses are given by (1 is the total evolution time):

$$\tau_1 = \frac{7}{10} \tau$$

The total Hamiltonian in the interaction picture (Tagging frame):

$$\tilde{H} = \tilde{H}_0 + \tilde{P} \tilde{Q} + \tilde{Q} \tilde{P}$$

where \( \tilde{Q} = \sum_j \langle j| \tilde{Q} |j\rangle \langle j| \) is the modified system Lindblad operator incorporating both the effects of the environment and the external control pulses.

The exact dynamics for the three-level quantum systems under both the environmental noise and control pulses can be compactly described by the non-Markovian QSD equation:

$$\frac{\partial}{\partial \tau} \tilde{P} = -i \left[ \tilde{H}, \tilde{P} \right] - \sum_j \lambda_j (\tilde{Q}_j \tilde{P} + \tilde{P} \tilde{Q}_j)$$

where \( \lambda_j \) represents the environmental bandwidth. By calculating the statistical average over many realizations of trajectory generated by the stochastic process, one can recover the density operator of the three-level system:

$$\rho = \tilde{\rho}_0 (\tilde{P}) = \tilde{\tilde{\rho}} (\tilde{P}, \tilde{Q}, \tilde{P})$$

Comparing the numerical results illustrated in the three pictures of Fig. 1, we see that the UDD control scheme can well protect the initial state, and a better protection result can be achieved by increasing the number of control pulses.

From Fig. 2, we find the effectiveness of the UDD control is affected by the environment memory time scale. For a long environment memory time (small \( \lambda_2 \)), which represents a non-Markovian environment, a longer coherence time may be preserved.

The above dynamical decoupling control scheme works for arbitrary unknown qudit states. Fig. 3 shows the control results for mixed initial state. The parameter \( M \in [0,1] \) describes the “degree of mixture”.

Conclusion

- Optimal control sequences for different noises environment.

Future Work

- Dynamic Decoupling is a promising strategy to control an unknown quantum state.
- QSD equation is very efficient in dealing with complicated open quantum dynamical problems.
- Entanglement control between qudit-system.
- Non-ideal pulses dynamics for UDD scheme.
- Optimal control sequences for different noises environment.

Reference