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Word Measures on Unitary Groups

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11:30am

Abstract:
We study measures induced by free words on the unitary groups $U(n)$: let w be a word in the free group F_r on r generators $x_1, \ldots, x_r$. For every $i = 1, ..., r$ substitute x_i with an independent, Haar-distributed random element of $U(n)$ and evaluate the product defined by w to obtain a random element in $U(n)$. The measure of this element is called the w-measure on $U(n)$.

Let $Tr_w(n)$ denote the expected trace of a random unitary matrix sampled from $U(n)$ according to the w-measure. It was shown by Voiculescu (91') that for $w \neq 1$, this expected trace is $o(n)$ asymptotically in n. We relate the numbers $Tr_w(n)$ to the theory of commutator length of words and obtain a much stronger statement. Our analysis also sheds new light on the solutions of the equation $[u_1, v_1]...[u_g, v_g] = w$ in free groups. I will also present some interesting related open problems.

Based on joint work with Michael Magee.