Manhattan Algebra Day

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Equations in groups and Hilbert's tenth problem

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CUNY Graduate Center, Room 9205-04
10:30am

Abstract:
In this talk I will discuss the classical Diophantine problem (decidability of finite systems of equations) for a given structure $\mathcal{A}$. The Diophantine problem for $\mathbb{Z}$, i.e., the famous Hibert's tenth problem, is undecidable (Matiyasevich, 1970). However, the Diophantine problem for $\mathbb{Q}$ is a major open problem in number theory. It is conjectured that the Diophantine problem for an arbitrary ring of algebraic integers $\mathcal{O}$, known also as the generalized Hilbert's tenth problem, is undecidable. This conjecture was confirmed in some particular cases, but otherwise stands wide open for the last 45 years.

Our approach to Diophantine problems is based on a general notion of definability, or more generally, interpretability of one algebraic structure $\mathcal{A}$ by equations in another algebraic structure $\mathcal{M}$. These notions can be viewed as generalizations of algebraic and semi-algebraic sets in affine algebraic geometry. The crucial point here is that if $\mathcal{A}$ is interpretable by equations in $\mathcal{M}$ then the Diophantine problem in $\mathcal{A}$ effectively reduces to the Diophantine problem in $\mathcal{M}$.

We show that if a group G contains a maximal solvable subgroup B such that B is polycyclic and non-virtually-abelian, then there exists a ring of algebraic integers $\mathcal{O}$ definable by equations in G, which reduces the generalized tenth Hilbert's problem in $\mathcal{O}$ to the Diophantine problem in G. Examples of such groups G include finitely generated non-virtually-abelian nilpotent group, polycyclic non-virtually-abelian groups, and arithmetic groups that contain non-virtually-abelian solvable subgroups. Furthermore, we show that in some cases the inverse reduction from G to $\mathcal{O}$ holds as well, so the Diophantine problems in $\mathcal{O}$ and G are equivalent.

I will also discuss some other possible applications of interpretations by equations and highlight some related open problems.

Talk is based on a joint work with Albert Garreta and Alexei Miasnikov.