“Group Theory International”
Online Seminar

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“A combination theorem for affine tree-free groups"

Thursday, May 12, noon (New York Time)

Isometric actions on Λ-trees have been studied by several authors, including Morgan, Shalen, Chiswell, Bass, Kharlampovich, Miasnikov, Remeslennikov and Serbin. In particular Bass showed how isometric actions of (vertex) groups on Λ_0-trees can be combined to give an isometric action (on a $Z\times\Lambda_0$-tree) of the fundamental group of an associated graph of groups, provided certain compatibility conditions are met. Notably, the hyperbolic lengths of the embedded images $\alpha_{e(g)}$ and $\alpha_{\overline{e}(g)}$ of elements g of edge groups must match up.

Affine actions are actions by dilations: one requires $d(gx,gy) = a_g\, d(x,y)$ where a_g is an order-preserving group automorphism of Λ. In this talk we will show how certain combinations of groups can be equipped with an affine action on a Λ-tree. That is, if a graph of groups is given where the vertex groups have affine actions on Λ_0-trees, the fundamental group admits an affine action on a Λ-tree where $\Lambda = Z\times\Lambda_0$, provided certain compatibility conditions are satisfied. Focusing on the case of free actions, we show that a large class of one-relator HNN extensions of free groups admit free affine actions on Λ-trees. Such HNN extensions cannot typically act freely by isometries because of the requirement images $\alpha_{e(g)}$ and $\alpha_{\overline{e}(g)}$ have the same hyperbolic length.

Using recent work by various authors, we also show that groups that admit a free affine action on a Z^n-tree with no inverted line are locally quasiconvex and relatively hyperbolic with nilpotent parabolic subgroups; they therefore have solvable word, conjugacy and isomorphism problems.

Next presentation: TBA in Fall 2016

Algebraic Cryptography Center