An introduction to sofic structures

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A rough analogy

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The answer is of course yes—just consider, say, a torus

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Answer: not clear!

Sofic groups

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Definition (Gromov, 1999)

A finitely generated group G is **sofic** if for any $\varepsilon > 0$ and any r, there exists a finite graph Γ such that

$$\frac{|\{x \in \Gamma : B_r(\Gamma, x) \cong G_r\}|}{|\Gamma|} \geq 1 - \varepsilon.$$

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The adjective "sofic" does not come from Gromov but from Weiss, who gave sofic groups their name in 2000 (the word "sofic" is derived from the Hebrew word for "finite")

In fact the word "sofic" is not new to the mathematical literature: it also appears in topological dynamics (more specifically, the theory of subshifts of finite type), where it was also introduced by Weiss

And yes: there is a connection with sofic groups

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See the survey of Pestov, 2008, for much more

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Does there exist a nonsofic group?

The proverbial "expert opinion" seems to be that there should exist a nonsofic group (I have no idea one way or the other)

Alternative definitions

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Definition

Let G be a group, $K \subseteq G$ a subset, and $\varepsilon > 0$. A (K,ε)**-action** of G on a finite set X is a function $\psi : G \to \mathrm{Map}(X)$ such that

i. If $g, h, gh \in K$, then $d_{\mathrm{Ham}}\left(\psi(gh), \psi(g) \circ \psi(h)\right) \leq \varepsilon$
ii. If $e \in K$, then $d_{\mathrm{Ham}}(\psi(e), \mathrm{Id}_X) \leq \varepsilon$, where e is the group identity
iii. For all distinct $g, h \in K$, we have $d_{\mathrm{Ham}}(\psi(g), \psi(h)) \geq 1 - \varepsilon$

Alternative definitions, continued

Definition (Elek and Szabó, 2005)

A group G is **sofic** if for any finite subset $K \subseteq G$ and any $\varepsilon > 0$, it admits a (K, ε)-action on a finite set X.

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> However: an arbitrary group is sofic if and only if all of its finitely generated subgroups are sofic

Note that if G admits a (K, ε)-action on a finite set X, then $\mu_X =$ uniform probability measure on X is **almost invariant**, i.e.

$$\|\mu_X - \psi(g)_* \mu_X\| \leq \varepsilon'$$

for all $g \in K$

Alternative definitions, continued some more

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Definition

A group G is **sofic** if δ_G, the Dirac measure on the Cayley graph of G, is the weak-$*$ limit of unimodular measures that come from finite graphs.

Soficity for other structures

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Idea: if μ is a unimodular measure on $\mathcal{G}$, then for a given r, it charges a certain number of r-nbhds $B_1, \ldots, B_k$

To approximate μ at scale r, we need a finite graph such that the proportion of its vertices with r-nbhds $\cong B_i$ is close to $\mu(B_i)$

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Important fact: the space of unimodular measures is closed, therefore...

1) Any weak-$*$ limit of unimodular measures that come from finite graphs is unimodular
2) Hence, only makes sense to define soficity with respect to unimodular (invariant) measures

A nice result

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Unimodular random trees are sofic.

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Is every unimodular random graph sofic?

Again, the sofic question remains open...

Sofic random subgroups

Let $L(G) =$ lattice of subgroups of a group G (e.g. a free group)

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Theorem (C., 2014)

The boundary action of a sofic random subgroup of the free group is conservative.

Other structures

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Ceccherini-Silberstein and Coornaert, 2013, recently introduced **sofic monoids**

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- Sofic monoids (Ceccherini-Silberstein and Coornaert)

In each of the above contexts, the sofic question remains open and seems very difficult...except one:

Theorem (Ceccherini-Silberstein and Coornaert, 2013)

There exists a nonsofic monoid.

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Definition of a sofic monoid: take the definition of a sofic group in terms of (K, ε)-actions and replace the word "group" with the word "monoid"

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Ceccherini-Silberstein and Coornaert: B does not, in general, admit (K, ε)-actions on finite sets

A schism?

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This raises the question: can one develop an alternative theory, where invariance plays a role?

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He also asked:

Question (Kambites, 2014)

Is there "an alternative, probably stronger, definition of a sofic monoid which also generalises sofic groups but exerts more control on the internal structure of the rest of the monoid"?

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Is there "an alternative, probably stronger, definition of a sofic monoid which also generalises sofic groups but exerts more control on the internal structure of the rest of the monoid"?

Kambites goes on to speculate that "such a definition, if found, is also likely to extend naturally to semigroups without an identity element"

An idea

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Here's an attempt:

Definition

Say that a monoid M is **sofic** (in an invariant way) if for any finite $K \subseteq M$ and any $\varepsilon > 0$, it admits a (K, ε)-action on a finite set X such that $\mu_X =$ uniform probability measure on X is almost invariant:

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In fact, might as well extend this definition to semigroups

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For lack of a better term, say that a finitely generated semigroup S is **homogenous** if for any two $x, y \in S$ and any r, the r-nbhds of x and y in (any) Cayley graph of S are isomorphic.

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Proposition, C.

If a semigroup is sofic in the invariant sense, then it is homogenous.

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The bicyclic monoid is again nonsofic, but this time for obvious reasons: it is not homogenous (it is not an **invariant structure**)

The question of whether there exists a nonsofic (homogenous) semigroup would again seem to be quite difficult

Concluding thought

Let's return to the infinite, homogenous template—if such a template is nonunimodular, then it cannot be sofic: it cannot admit finite approximations

Example: the **grandfather graph** of Trofimov, 1985

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Thurston, 1982: a **geometric structure** is a space modeled on a homogenous space (X, G), where $X =$ manifold and $G =$ group of diffeomorphisms with compact stabilizers

If G is nonunimodular, then X possesses a G-invariant, volume-expanding vector field, which makes it impossible to find finite-volume geometric structures modeled on (X, G)

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It would be nice, of course, to have a better geometric understanding of obstructions to soficity in general!

Concluding thought, continued

So the same phenomenon appears in the world of manifolds and the world of graphs—it would be nice to have a better geometric understanding of it in the latter

It would be nice, of course, to have a better geometric understanding of obstructions to soficity in general!

What can go wrong when one tries to create finite models of infinite, homogenous templates?

Thank You!