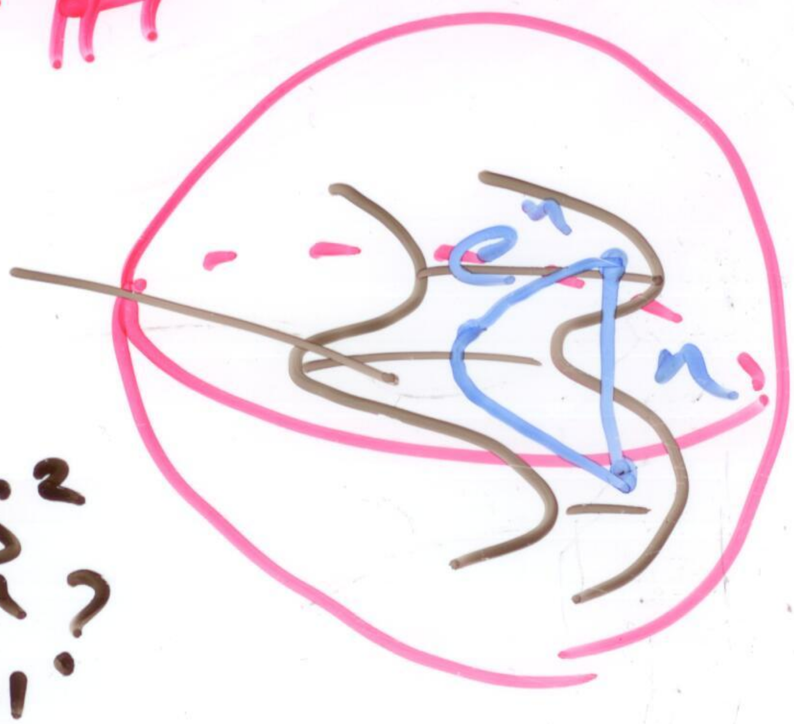
$(S \times I)/\sim\ = M$

pseudo-Anosov homeo

$\tilde{M} = \mathbb{H}^3$

$\tilde{S} = \mathbb{H}^2$



$\partial \tilde{M} = S^2$

?

$\partial \tilde{S} = S^1$

X a geodesic hyp space

$e \in X$

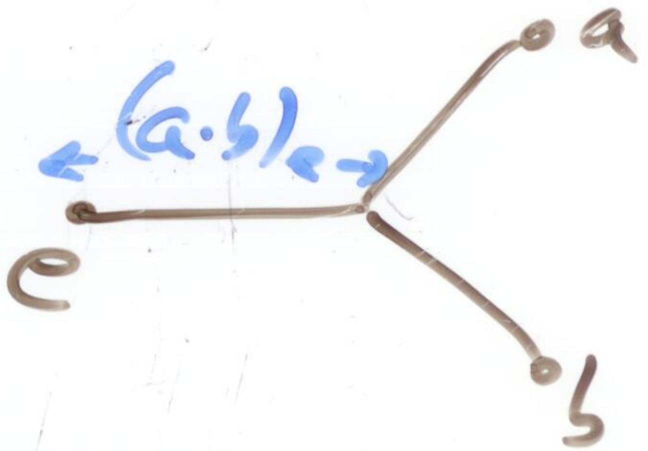
$$\partial X := \left\{ \begin{array}{c}\text{inf. geodesic rays} \\ \text{from } e\end{array} \right\} \Big/ \sim$$

$\gamma \sim \gamma'$ when

$$d(\gamma(t), \gamma'(t)) \leq \text{const} \quad \forall t$$

$$(a \cdot b)_e = \frac{1}{2}\left(d(a,e) + d(b,e) - d(a,b)\right)$$

$\simeq$ how far geodesics $[e,a]$ and $[e,b]$ fellow-travel



$\simeq d(e, [a,b])$

$$\partial X = \left\{ (a_n) \mid (a_m \cdot a_n)_e \to \infty \text{ as } m,n \to \infty \right\} / \sim$$

$(a_n) \sim (b_n)$ when $(a_m \cdot b_n)_e \to \infty$ as $m,n \to \infty$.

$(X, d_x), (Y, d_Y)$ both hyp.
with $e \in Y \subseteq X$

Cannon–Thurston Map

$$(a_n) \longmapsto (a_n)$$
$$\partial Y \longrightarrow \partial X$$

C–T Well defined in their example $S^1 \twoheadrightarrow S^2$
Space filling curve.

$H \leq G$ both hyp.

$\shortparallel \qquad \shortparallel$

$Y \qquad X$

Mitra

$H = \pi_1 S$

$G = \pi_1 M$

N. Brady

Hyperbolic ~~Hydra~~, Dison - R

$G_k = F(a_0, a_1, \dots, a_k, b_1, \dots, b_\ell) \rtimes \mathbb{Z}$ "$\langle t \rangle$"

Γ_k =

$t^{-1} a_k t = a_k a_{k-1}$

$\vdots$

$t^{-1} a_2 t = a_2 a_1$

$t^{-1} a_1 t = a_0$

$t^{-1} a_0 t = u a_1 v$

u, v words on the b's

$t^{-1} b_i t = b_*$

$H_k = \langle a_0 t, b_1, \dots, b_\ell, a_1 t, \dots, a_k t \rangle \cong F_{k+1+\ell}$ hyp.

Λ_k =

$\mathrm{Dist}_{\cancel{H_k}\ \Lambda_k}^{\cancel{G_k}\ \Gamma_k} \succeq A_k$

$A_1(n) = 2n$

$A_2(n) = 2^n$

$A_3(n) = 2^{2^{\cdot^{\cdot^{2}}}} \Big\} n$

$\vdots$

Hydra battle

$$a_3 a_3 a_3$$

$$a_3 a_2 a_3 a_2$$

$$a_2 a_1 a_3 a_2 a_2 a_1$$

$$\vdots$$

$$\varepsilon$$

Duration against $a_k^n \simeq A_k(n)$

$$a_k^n a_2 t a_1 a_2^{-1} a_k^{-n} \in H_k$$

$\underline{\text{Th}^{\underline{m}}}$ The C–T map for $\Lambda_k \leq \Gamma_k$ is well-defined.

Modulus of Continuity

$f : U \to V$, $\delta > 0$

$$\varepsilon(\delta) := \sup \{ d(f(a), f(b)) \mid a, b \in U \text{ with } d(a,b) \leq \delta \}$$

"Visual metric":

$$d(p,q) \sim e^{-(p \cdot q)_e}$$

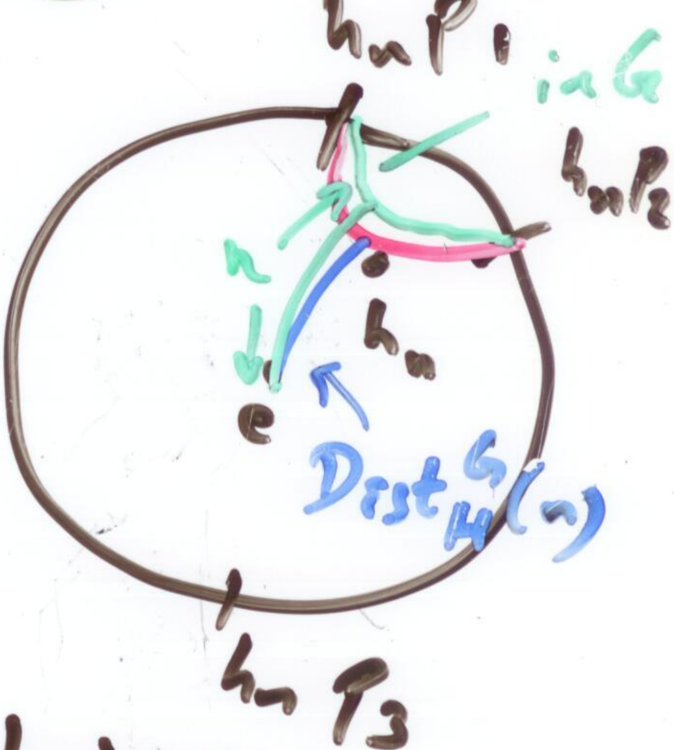
Th$^{\underline{m}}$ $H \leq G$ both hyp

If $\partial H \to \partial G$ is well-def.

and H is not elementary, then

$\exists\, \alpha, \beta > 0$, $\forall n$,

$$\mathcal{E}\left(\frac{\beta}{e^{\mathrm{Dist}_H^G(n)}}\right) \geq \frac{\alpha}{e^n}$$



$h_n \in H$ with $d_G(e, h_n) \leq n$

and $d_H(e, h_n) = \mathrm{Dist}_H^G(n)$.

$\underline{\mathrm{Th}^{\underline{m}}}$ Let

$$G = \left\langle a, b, c_1, c_2, d_1, d_2 \;\middle|\; \begin{array}{l} a^{-1}b^{-1}ab = C \\ b^{-1}c_i b = C_i \\ (ab)^{-1} d_j ab = D_j \\ c_i^{-1} d_j c_i = D_{ij} \end{array} \right\rangle$$

$\exists$ +ve words C, C_i on c_1, c_2 and D_j, D_{ij} on d_1, d_2 with

(i) G hyp

(ii) $H := \langle b, d_1, d_2 \rangle \cong F_3$

(iii) $\partial H \to \partial G$ __not__ well-defined.

(i) Small cancellation

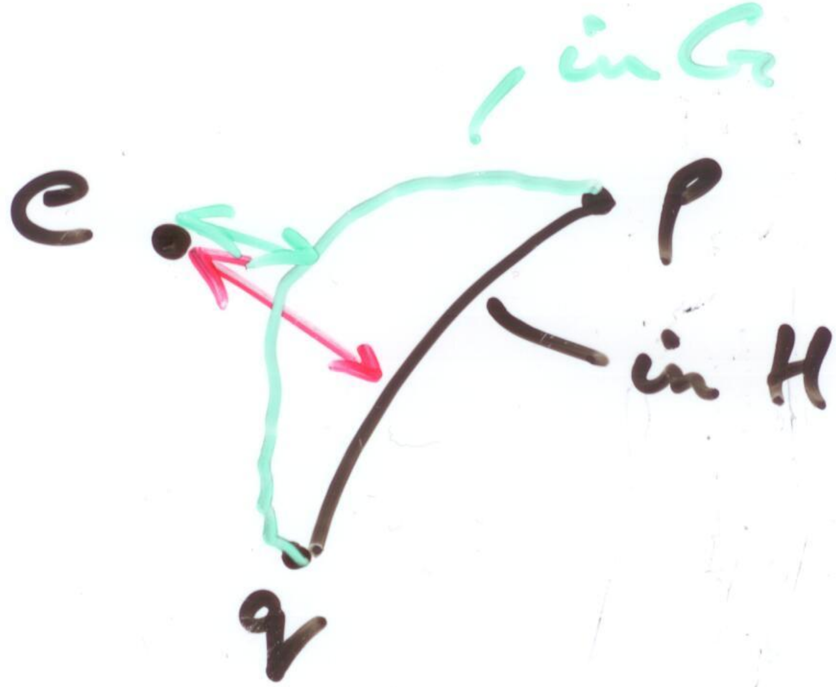
(ii) HNN- structure

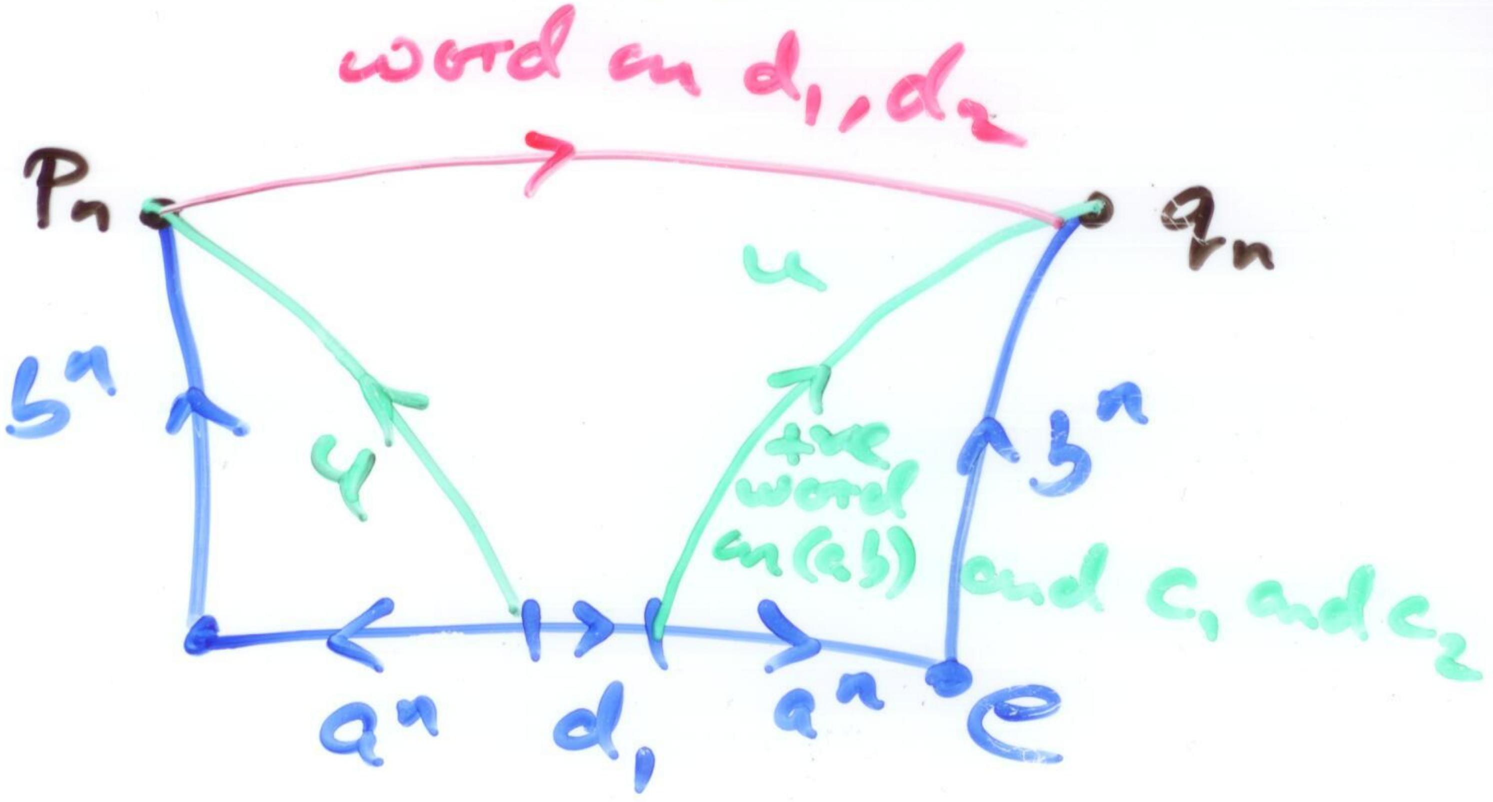
(iii) Mitra's Lemma

C-T map is well defined

$\Leftrightarrow (p\cdot q)^G_e \to \infty$ as

$(p\cdot q)^H_e \to \infty$ for $p,q \in H$.





$$(p_n \cdot q_n)^H_e = n$$

$$(p_n \cdot q_n)^G_e \sim 0$$