Group Theory on the Hudson

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"Actions, length functions, and non-Archimedean words"

Abstract:

In 2013 the theory of group actions on Λ–trees will be half a century old. There are three stages in the development of the theory: the initial period when the basic concepts, methods and open problem were laid down; the period of concentration on actions on R-trees; and the recent phase when the focus is mostly on non-Archimedean actions.

The first definition of an R-tree appeared in the work of Tits, 1977, in connection with Bruhat-Tits buildings. One year earlier Chiswell, 1976, came up with a crucial construction that shows that a group G with a Lyndon length function $l : G \rightarrow R$ has a natural action by isometries on an R-tree (even though he did not refer to the space as an R-tree), and vice versa. This was an important result which showed that group actions and Lyndon length functions are just two equivalent languages describing essentially the same objects. In 1984, Morgan and Shalen introduced Λ–trees for an arbitrary ordered abelian group Λ and established a general form of Chiswell's construction. In the subsequent papers Morgan and Shalen, and Culler and Morgan further developed the theory of R-trees and group actions. In 1987, Alperin and Bass developed much of the initial framework of the theory of group actions on Λ–trees and stated the Fundamental Problem: *Find the group theoretic information carried by an action on Λ–tree, analogous to the Bass-Serre theory for the case $\Lambda = Z$.*
It is not surprising, from the view-point of Bass-Serre theory, that the following problem became of crucial importance:
The Main Problem: Describe f.g. (f.p.) groups acting freely on Λ–trees.

I am going to survey the subsequent development of the area and, in particular, our results. We give a solution of Alperin and Bass problem for f.p. groups, and some problems from Chiswell's book, prove that every f.p. Λ-free group G is R^n-free, toral relatively hyperbolic, has a quasi convex hierarchy, virtually special (that is, some subgroup of finite index in G embeds into a right-angled Artin group), virtually orderable, linear, etc.